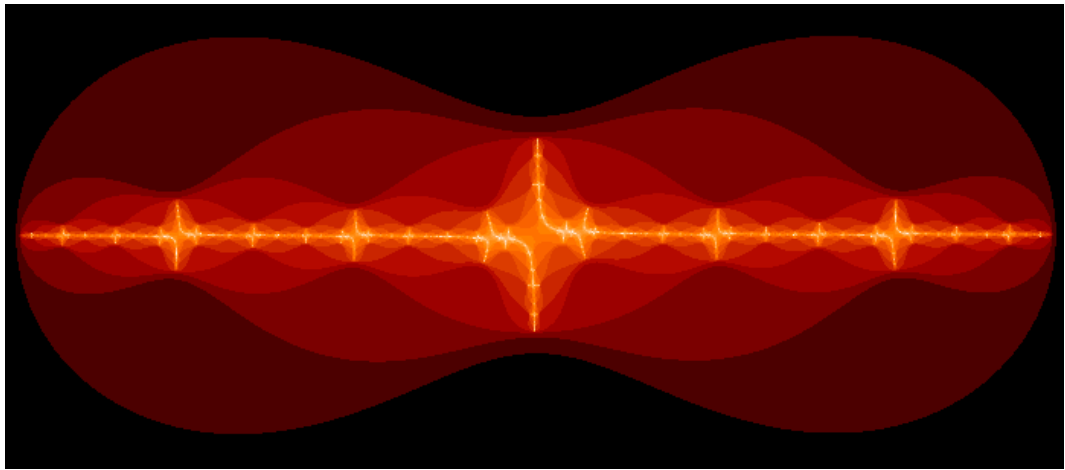


TigerGraphics

MATHTOOLS



Educational Tool for applied Maths

Version **1.4**

User Manual

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1 Introduction

MATHTOOLS is an educational tool for the visualization of several nice mathematical things.

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MATHTOOLS is built on the base of `Tcl/Tk` version **8.6** and `Gnu-C` version **7.5**. It is developed and tested on OpenSuSE Linux Version **15.4** and is ported to run also on Microsoft Windows and MacOS platforms, where 64-bit architectures are assumed.

1.1 Linux Installation

1.1.1 Requirements

Make sure that at least `gcc`, `make`, `tcl` and `tk` are installed on your system.

Furthermore make sure that **Python** at least in the version **3.10** with package **SymPy** is available on your system, either by system wide installation or locally for your user, which is supported by **Python**. Please read the according **Python** installation information.

Python is required for the linearisation, normalization of differential equation systems and for isocline computation in the `Slope` module.

1. Copy the file `mathtools.tgz` to the directory where the software should be installed.

Extract the data from the `.tgz`-file by

```
tar -zxvf mathtools.tgz
```

The archive is unpacked to the directory `MATHTOOLS`.

2. Compiling

The MathTools are running on 32-bit and 64-bit systems. Thus, the binaries need to be built after unpacking. Go to the directory `MATHTOOLS/src` by f.e.

```
cd MATHTOOLS/src
```

and execute the script `install_all` from the commandline

```
./install_all
```

All the Makefiles in the subdirectories will be executed.

1.1.2 Starting

Go to the directory `MATHTOOLS`. The main script `mft.tcl` is executable from the commandline and can be referenced by any application starter.

1.2 Windows Installation

Execute the file `mathtools-setup.exe` and follow the instructions. The program can be started from the menu in the execution group `TigerGraphics` or from the desktop if the option for a desktop icon was selected.

If you already have **Python** with **SymPy** installed just set the path to the **Python** executable referring to **[Custom]** (2.1).

If don't want to or cannot install **Python** please download the alternative installer `mathtools_withpy-setup.exe`.

It is quite larger but a suitable version of **Python** is included with the installer and the path to **Python** is already set.

1.3 Mac OS Installation

Download the appropriate application for your architecture (`x86` or `arm`). Finally, copy the application container `MFT.app` either to the systems application folder `Programs` or to any other desired location and simply start the application like any other one.

Furthermore make sure that **Python** at least in the version **3.10** with package **SymPy** is installed on your system.

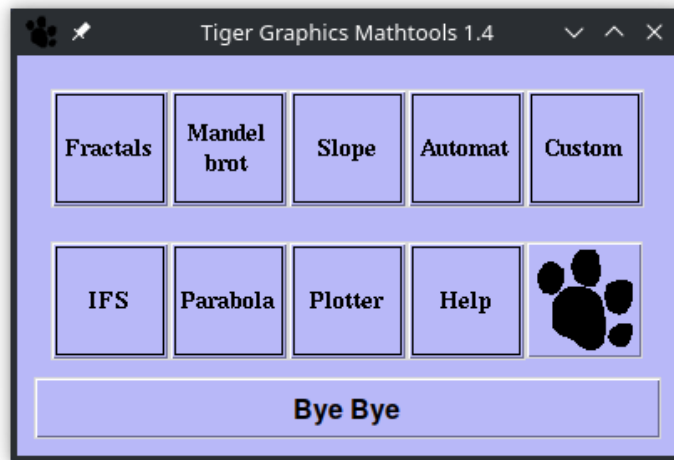
Please read the according **Python** installation information from <https://python.org>.

1.4 Python Licence

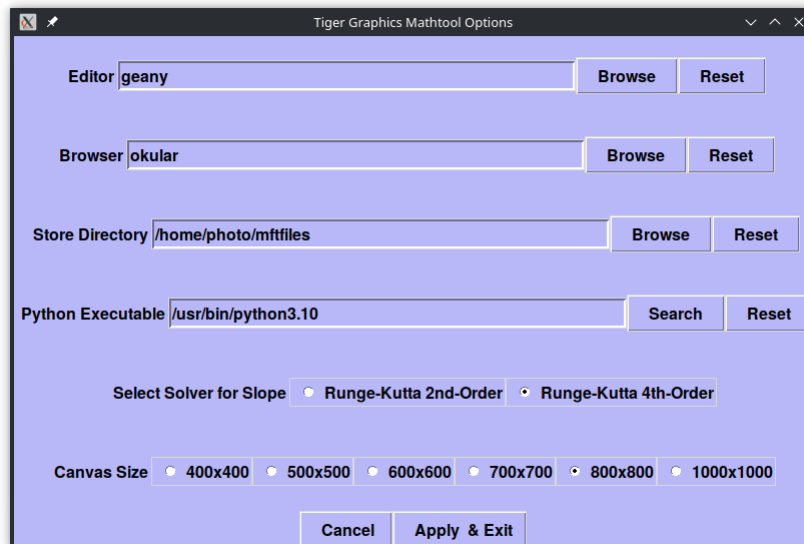
We refer to the PYTHON SOFTWARE FOUNDATION LICENSE VERSION 2 as published at <https://docs.python.org/3/license.html>.

2 MATHTOOLS Main Widget

The **MATHTOOLS** main widget serves to switch to the different groups of tools described in the following sections.



2.1 Custom



The default editor and the default browser for the **MATHTOOLS** Help can be set. In the entry **[Store Directory]** a directory shall be selected according to your preferences where **MATHTOOLS** will store plots, model files and from where the latter ones could be loaded. Also the path to the **Python** executable must be set here. Please note that at least `Python 3.10` is required for symbolic analyses in the **[Slope]** tool. To calculate trajectories in the **[Slope]** tool one of two Runge-Kutta solvers can be selected as the 'prime' solver. Please consider the speed of your computer – Runge-Kutta 2nd-Order is faster.

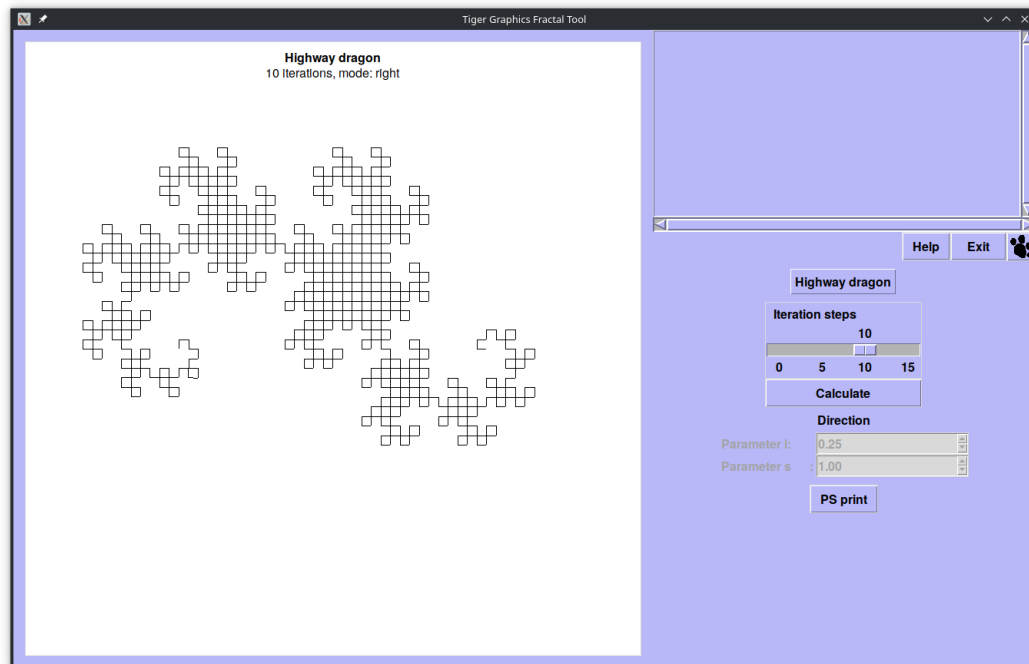
Remark: There are some defaults set. Such it should not be necessary to customize the settings. If an error message occurs while trying to edit, browse, store or load please select the executables and settings according to your system!

2.2 Fractals

Several self similar fractals can be constructed:

- Highway dragon
- Koch Curve
- Cantor Set
- Weierstrass Monster
- Sierpinski triangle
- Pythagoras Tree
- Sunflower

The number of iteration steps can be set by the slider. The construction laws are self-explanatory if a low number of iteration steps is selected.



2.2.1 [Calculate]

The number of selected iterations is calculated and displayed.

Parameter settings

Highway dragon The direction of the construction can be selected. The effect can be seen if only a few iterations are selected.

Weierstrass monster The limit object of the Weierstrass Monster is a function which is continuous in \mathbf{R} , but not differentiable at any point in \mathbf{R} . It is defined by

$$f(x) = \sum_{k=0}^N \lambda^{k(s-1)} \cdot \sin(\lambda^k t)$$

where N is the number of iterations. The values for λ and s can be selected.

Note: not every parameter combination leads to nice monsters.

The fractal dimension of the monster is estimated and displayed. This is only a very rough estimation, because the calculation is done on the base of the Nth iteration by the curve length.

Sunflower The configuration of seeds in a sunflower is simulated. Starting from a central point the radius of every step is increased by s while the angle is increased by φ given in degree.

2.2.2 PS print/Print

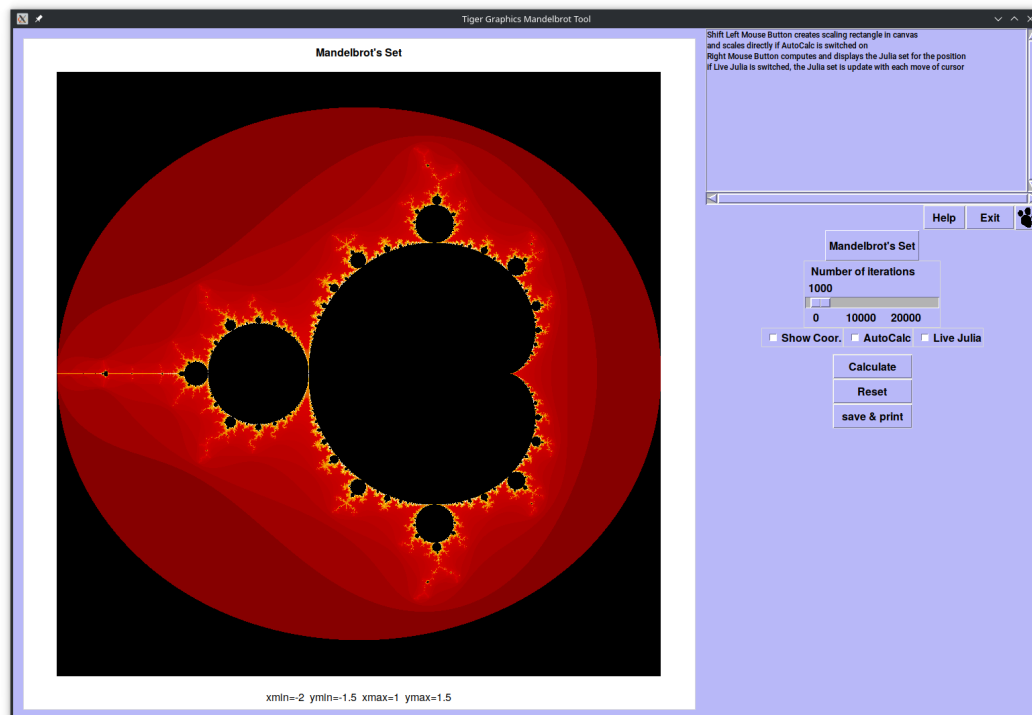
A Postscript output is generated and stored in

<Path to Store Directory>/plots. The PS-files are named automatically.

2.3 Mandelbrot

This tool collection included three element: the classical Mandelbrot set, the attractors of solving the complex equation $x^3 = -1$ by Newton's method and the so called Collatz fractal.

2.3.1 Mandelbrot Set



The Mandelbrot set is constructed and displayed. The Mandelbrot set is defined as the following subset of the complex plane:

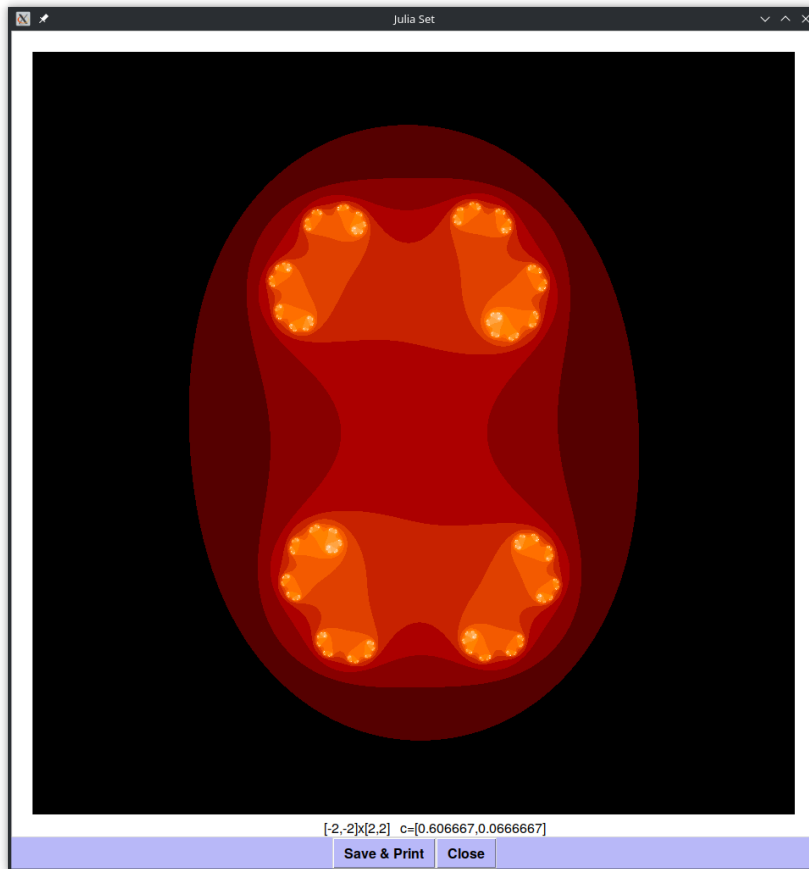
$$M = \{c \in \mathbb{C} \mid (x_n) \text{ remains bounded, } x_{n+1} = x_n^2 + c, x_0 = c\}$$

Such, in the picture, the black object is the Mandelbrot set. The nice colors are determined by the speed of divergence.

The Julia set of a point is calculated if clicked with the right mouse button and additionally by selection the LiveJulia checkbox (see below). Clicking the right button at a point in the image initiates the computation of the Julia set with the selected number of iterations and stops the LiveJulia function.

The Julia set of a given point c is defined as the following set of points in the complex plane:

$$J_c = \{x \in \mathbb{C} \mid (x_n) \text{ remains bounded, } x_{n+1} = x_n^2 + c, x_0 = x\}$$



Live Julia

Selecting this checkbox opens a new window. In this new window the Julia set of the Mandelbrot fractal is live updated upon cursor movements. To be fast enough, the Julia set is computed with a reduced number of iterations.

2.3.2 Newton roots

The tool allows also the visualization of the basins of the roots z_i of the complex equation $x^3 = -1$ calculated by Newton's method.

The three basins B_i are defined as the following sets of points:

$$B_i = \{x \in \mathbb{C} \mid x_n \rightarrow z_i, x_0 = x, x_{n+1} = x_n - \frac{f(x)}{f'(x)}, f(x) = x^3 + 1\}$$

2.3.3 Collatz' Fractal

The Collatz fractal in the complex plane is derived from the Collatz conjecture, a mathematical sequence defined by the function:

$$f(z) = \begin{cases} \frac{z}{2} & \text{if } z \text{ is even} \\ 3z + 1 & \text{if } z \text{ is odd} \end{cases}$$

or in complex formulation ¹:

$$f(z) = \frac{2.0 + 7.0 \cdot z - (2.0 + 5.0 \cdot z) \cdot \cos(\pi \cdot z)}{4.0}$$

Starting with a complex number z_0 , the Collatz fractal is generated by iteratively applying the function $f(z)$ to the result of the previous iteration. The behavior of this iteration is observed in the complex plane.

2.3.4 Common canvas functions

Within the picture (the canvas area) a rectangle can be defined by pressing and holding the left mouse button and dragging the mouse while the button is pressed. **[Calculate]** starts the calculation of the new cut-out. The number of iterations must be set higher the more you go into detail. Additionally, you can zoom in and out by using the mouse wheel.

2.3.5 Reset

Pressing this button resets the canvas to the initial coordinates. The selected number of iterations is not effected.

2.3.6 Calculate

The part of the Mandelbrot Set is calculated depending on the cut-out. The number of iterations determines the accuracy.

2.3.7 Reset

Pressing this button resets the canvas to the initial coordinates. The selected number of iterations is not effected.

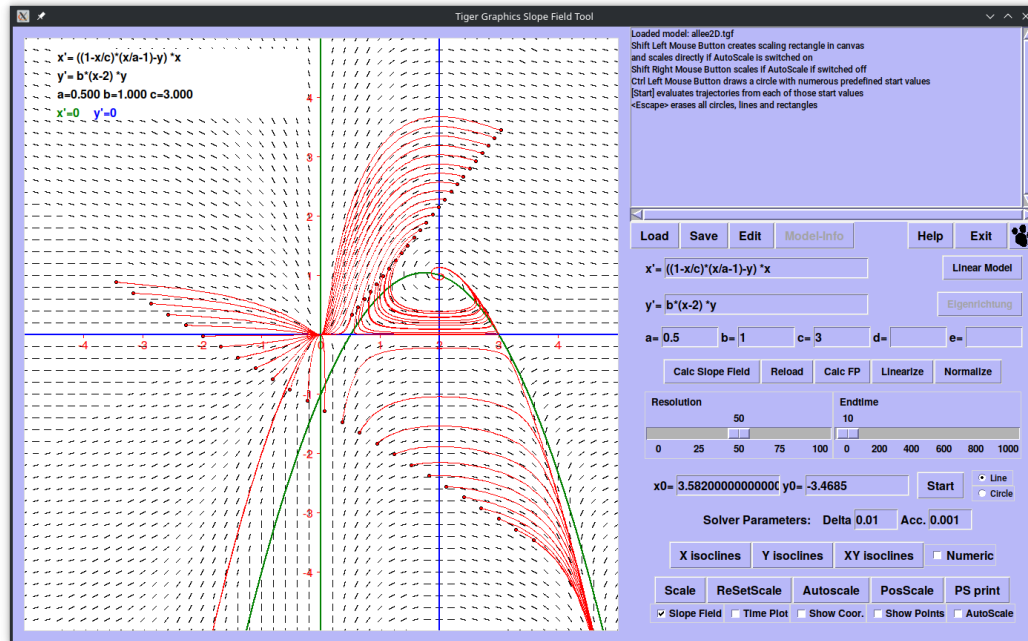
2.3.8 Save & Print

A Postscript output and a gif output is generated and stored in `<Path to Mathtools>/mftfiles/plots`. The files are named automatically.

¹The derivation of this representation can be found at <http://www.3d-meier.de/tut20/Collatz/Seite1>

2.4 Slope

This tool allows the visualization of simple one or two dimensional differential equations.



2.4.1 One Dimensional differential equation

Set $x' = 1$. This means that $x(t) = t$ ($t_0 = 0$). Set $y' =$ to the differential equation which should be considered. **[Calc Slope Field]** computes and displays the slope field of the differential equation in the t - y plane.

2.4.2 Two Dimensional differential equation

Set $x' =$ and $y' =$ to your differential equation system which should be considered. **[Calc Slope Field]** shows the slope field of the system in the x - y phase plane.

2.4.3 Parameters and Math-Built in functions

Up to five parameters (a, b, c, d, e) can be used within the formulas.

The following math functions can be used within the formulas:

```
acos asin atan cos cosh exp log log10
pow sin sinh sqrt tan tanh abs
```

2.4.4 Linear Model

If the parameters a, b, c, d are set **[linear]** makes a linear system from these parameters:

$$\begin{pmatrix} x' \\ y' \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix}$$

The Eigenvalues und Eigenvectors, the trace and determinant are displayed in the Info window.

2.4.5 Eigenrichtung

The directions of the Eigenvectors are plotted. This button is only active if we have non-complex Eigenvectors.

2.4.6 Reload

Pressing this button forces a reload of the original differential equation or differential equation system when needed, especially after the derived linear model has been computed and you want to return the non-linear.

Python dependent functions

The functions **[Calc FP]** , **[Make Linear]** and **[Normalize]** require **Python** for symbolic analyses. If no suitable **Python** is found (see **[Custom]** (2.1)), those buttons will not appear.

2.4.7 Calc FP

The fixed points of the the defined system are computed and displayed in the Info window.

2.4.8 Make Linear

The linear model is derived and the fixed points are derived. If there are more than one fixed point, a selection window is opened to select one of the fixed points described by its value. After selection the according slope field is computed and displayed.

After the linear model has been derived, those input fields and function buttons which are meaningless for the derived linear model are disabled. They all are enabled again after reloading the original equations by **[Reload]** or loading a new model by **[Load]** .

2.4.9 Normalize

Normalizing serves to compare models of different types by unifying the scales and shifting the fixed points to $[1, 1]$ in the $[x, y]$ -plane. The process is as follows for generic model:

Let the model be described by:

$$\dot{x} = (g(x) - y \cdot h(x)) \cdot x \quad \dot{y} = a \cdot (x \cdot h(x) - m) \cdot y$$

For a fixed point (x^*, y^*) with $x^* > 0$ and $y^* > 0$

the normalized model is derived by the substitution $X=x/x^*$ and $Y=y/y^*$

$$\begin{aligned} \dot{X} &= \frac{dX}{dt} = \frac{1}{x^*} \cdot \frac{dx}{dt} = (g(X \cdot x^*) - Y \cdot y^* \cdot h(X \cdot x^*)) \cdot X \\ \dot{Y} &= \frac{dY}{dt} = \frac{1}{y^*} \cdot \frac{dy}{dt} = a \cdot (X \cdot x^* \cdot h(X \cdot x^*) - m) \cdot Y \end{aligned}$$

The formulas in the function entries are replaced by those for the normalized model where x stands for the term $(X \cdot x^*)$ stands and y stands for the term $(Y \cdot y^*)$; the fixed point coordinates are denoted by x_f and y_f .

If none or only one considerable fixed point is found, a message is given in the info window; in case of more than one considerable fixed points are found a selection window is opened to select one of the fixed points described by its value. After selection the according slope field is computed and displayed.

If at least one considerable fixed point is found, the slope field of the normalized model is computed and displayed.

After the linear or a normalized model has been derived, those input fields and function buttons which are meaningless for the derived model are disabled. They all are enabled again after reloading the original equations by **[Reload]** or loading a new model by **[Load]**.

2.4.10 Resolution

The slider **[Resolution]** sets the resolution of the slope field. After setting a new value the slope field is refreshed.

2.4.11 Endtime

The slider **[Endtime]** sets the endtime for the evaluation of the trajectories.

2.4.12 Isoclines

The isoclines of the system ($x' = 0$, $y' = 0$) can be plotted.
Either the single isoclines **[X isoclines]** or **[Y isoclines]** or both with **[XY Isoclines]**. If **Python** is available the isoclines will be derived by symbolic analysis otherwise they will be determined numerically, which is less accurate.

2.4.13 [Scale]

The x and y ranges of the plotting area for the slope field can be set. Applying the input values by **[Apply]** refreshes the slope field; **[Cancel]** just closes the window and rejects the changes.

2.4.14 [ReSetScale]

This button resets the the range of the plotting area to the initial values.

2.4.15 [Autoscale]

The maximum values of x and y are evaluated and the plotting area is adapted to those values accordingly.

2.4.16 [PosScale]

The x and y range is shifted to the positive quadrant.

2.4.17 Trajectories

Trajectories to given initial values can be plotted in two ways:

1. Setting initial values for x and y in $x_0 =$ and $y_0 =$ and pressing **[Start]** or
2. Setting an initial value by a left mouse click within the slope field. In this case it is helpful to switch on **[Show coor.]** to see the coordinates of the mouse cursor.

The calculation is done by evaluating the differential equation with a second order method. By middle mouse click the evaluation is done with Euler's method. The time step for the evaluation is $\Delta t = 0.01$.

The simulation stops either at **[Endtime]** or can be stopped by a right mouse click in the slope field.

2.4.18 Multi-Trajectories

Although trajectories can be started from any point in the slope field, it might be uncomfortable to inspect areas of interest by a lot of single clicks.

On the right hand side of the **[Start]** button you can select whether to draw a line or a circle for defining a set of initial values for trajectories.

By holding the <CTRL> key and pressing the left mouse button a line or circle can be drawn from the point of origin by dragging the mouse pointer while keeping <CTRL> pressed.

After releasing the mouse button the line resp. the circle will be transformed into a set of initial values displayed as a set of blue dots.

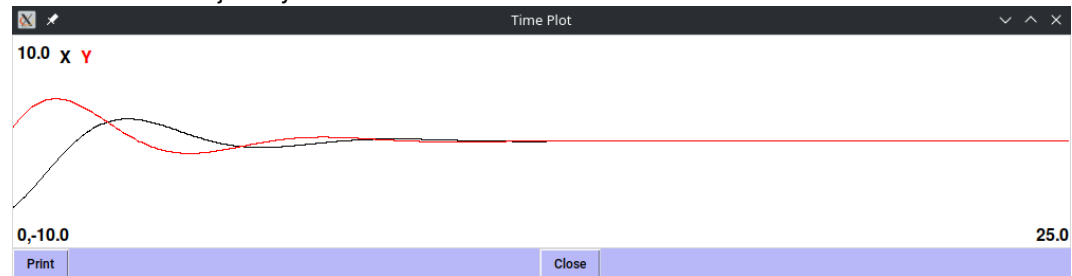
The number of dots is the half values of the **[Resolution]** parameter.

After pressing **[Start]** the evaluation of the trajectories for each of those initial points begins.

If a set of initial values is drawn but should not apply, it can be erased by pressing the <Escape>-Button on the keyboard.

Time Plot

In the case of a two dimensional system the time information cannot be seen in the phase diagram. If **[Time Plot]** is switched on a second plot window is opened showing the temporal evolution of the trajectory.



Show Coord.

If **[Show Coord.]** is switched on the coordinates in the slope field are shown at the mouse pointer.

Show Points

If **[Show Points]** is switched on a dot is displayed in the slope field after each integration step of the trajectory. The distance of those dots depend on the value of Δ .

AutoScale

If **[AutoScale]** is switched on the definition of a scaling/zooming area by Shift-Left Mouse Button is directly applied.

Otherwise Shift-Right Mouse Button will initiate the scaling.

If a scaling rectangle is drawn but should not apply, it can be erased by pressing the <Escape>-Button on the keyboard.

2.4.19 [Load] , [Save] & [Edit]

Equations, parameter values, coordinates, resolution and endtime can be saved in a file.

These files get automatically the extension `.tgf` and are stored normally in

<Path to Mathtools>/mftfiles. After loading a file the equations, parameter values and the stored coordinate values, resolution and endtime are set.

The files can also be edited to set f.e. coordinate values not provided by default. This editing functionality does not work under windows at the moment. Please use your usual editor to edit the files manually.

2.4.20 PS print/Print

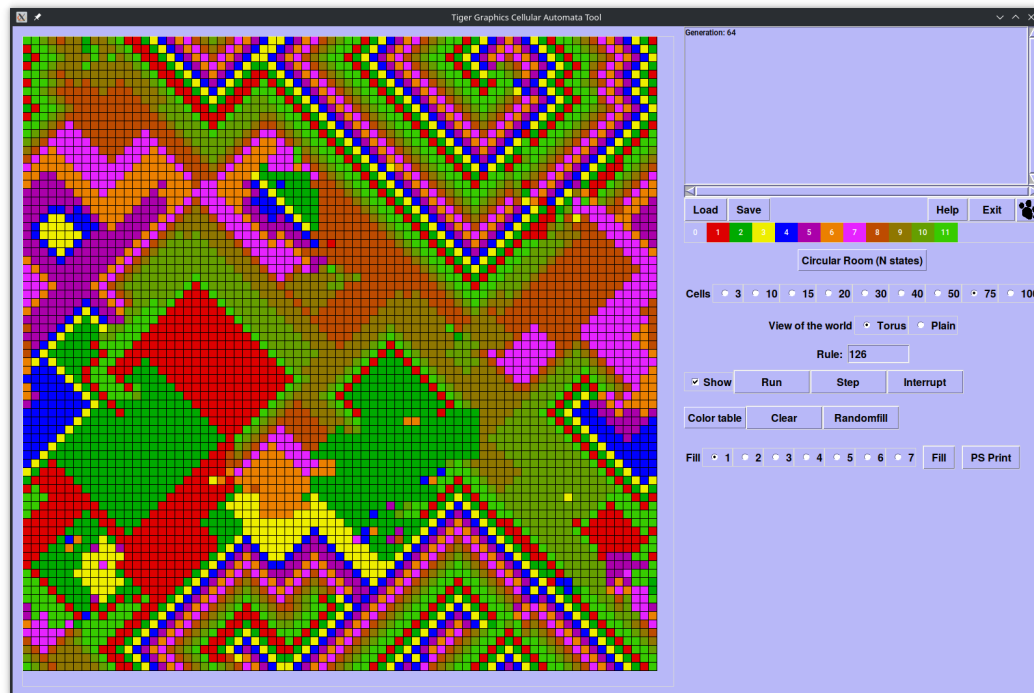
A Postscript output is generated and stored in
<Path to Store Directory>/plots. The PS-files are named automatically.

2.5 Cellular Automata

Cellular automata are discrete dynamical systems. Space is represented by a uniform grid, with each cell containing a value; time advances in discrete steps and the rules of change of each cell is determined by the states of its closest neighbour cells. Different cellular automata are implemented and can be simulated

- Conway's Life
- Wolfram Universe
- Disease Spread
- three implementations of Per Bak's sand piles
- two Circular rooms
- Spreading Bugs
- Diffusion
- three advection automats
- two diffusion based to generate leopard skins

The automaton can be selected by the pull down menu. The active one is displayed.



2.5.1 Life

Life was devised in 1970 by John Horton Conway. The automaton knows two states: dead (0, off) or alive (1, on). The neighbourhood of a cell is defined by the eight surrounding cells.

The rules

- if, for a given cell, the number of neighbours is exactly two, the cell maintains its status quo into the next generation. If the cell is on, it stays on, if it is off, it stays off.
- if the number of on neighbours is exactly three, the cell will be on in the next generation. This is regardless of the cell's current state.
- if the number of on neighbours is smaller or equal 1 or greater or equal 4 the cell will be off in the next generation.

2.5.2 Universe according to Stephan Wolfram

To understand the principle, we will first look at Stephen Wolfram's one-dimensional model. Stephen Wolfram (*1959) is a British physicist. He is the developer of Mathematica and the search engine Wolfram Alpha. His book "A New Kind of Science (2002)" describes many applications of cellular automata.

A simple example is the following automaton: We divide the space into a number of cells (e.g. 150). Each cell can assume the state dead or alive (0 or 1). Now you can start with a live cell in the centre.

The constellation in the next step results from the following rule

- A cell receives the new state 1 if there were one or two living cells in the block of three consisting of itself and its two neighbours.
- A cell receives the new state 0 if there were zero or 3 living cells in the block of three consisting of itself and its two neighbours.

If the last cell is reattached to the first (ring), the border cells also have 2 neighbours. You can then calculate more generations without getting edge effects.

The rules can be formalised as follows

1	1	1	1	1	0	1	0	1	1	0	0	0	1	1	0	1	0	0	0	1	0	0	0	0
		↓			↓			↓			↓			↓			↓			↓			↓	
		0			1			1			1			1			1			1			0	

You can now come up with many rules, each of which is uniquely represented by an eight-digit binary number if you agree on the order of the triplets. Wolfram used the above sequence and converted the eight-digit binary number into a decimal number. This allowed him to simply "name" all possible rules from 0 to 255. There are therefore 256 rule sets in total. The above rule is then number 126 (binary 01111110).

2.5.3 Disease

This automaton simulates the spread of a disease. A cell can either be susceptible (0), infected (1), ill (2) or immune (3). The automaton is not fully deterministic !!

The rules

- if a cell is susceptible it will be infected with a certain probability depending on the number of infected neighbours. The cell will be infected if the number of neighbours multiplied with a random number between 0 and 1 exceeds 0.9.
- infected cells become ill with a probability of 60%
- ill cells stay ill (50% probability), become immune (25% probability) or susceptible (25% probability)
- immune cell become susceptible with a probability of 5% to represent deaths and births

2.5.4 Per Bak's sandpile model

Per Bak's sandpile model is an example of self-organized criticality. It is a cellular automaton whose configuration is determined by the number of sandgrains on a cell.

The rules

- a grain of sand is added at a randomly selected cell
- a cell with more than 3 sand grains becomes unstable and topples by distributing one grain of sand to each of its four neighbours. This may cause unstable cells in the neighbourhood. An avalanche is born.
- if any avalanche dies out, another grain of sand is added to a randomly selected cell.

There are two models implemented:

- 4 neighbours. A cell with more than 3 sand grains becomes unstable and topples by distributing one grain of sand to each of its eight neighbours.
- the same model but with 8 neighbours. A cell with more than 7 sand grains becomes unstable and topples by distributing one grain of sand to each of its eight neighbours.

2.5.5 Circular Room

The circular room is a nice tool to demonstrate self organization. It is most impressive to start from a randomized initial state.

The rules

- The N states are circularly arranged: The state N is identified with state 0. Each cell has 4 neighbours.
- The state of a cell is increased by one if at least one neighbour has the state of the cell plus 1.

The Circular Room is implemented for 6 and 20 states.

2.5.6 Bug Spread

This is an example of an automaton combined with a differential equation model. There are two state variables, the number of bugs B within the part of the forest (cell) and the mean age A of the trees in this part of the forest. The differential equations for the bug growth, the aging of the forest and the bug spread are given by

$$\begin{aligned}\frac{\partial B}{\partial t} &= r \cdot A \cdot \left(1 - \frac{B}{K}\right) \cdot B - \mu \cdot \frac{B^2}{B^2 + M^2} + D_a \frac{\partial^2 B}{\partial x^2} \\ \frac{\partial A}{\partial t} &= \alpha \cdot (1 - A) - \beta \cdot \left(\frac{B}{K}\right)^3 \cdot A\end{aligned}$$

The diffusive spread is realized by an automaton scheme with an additional wind direction from west to east.

$$\begin{aligned}\Delta B(i, j) &= -4 \varepsilon_0 \cdot B(i, j) \cdot \Delta t \\ \Delta B(i-1, j) &= \varepsilon_0 \cdot (1 - \varepsilon_r) \cdot B(i, j) \cdot \Delta t \\ \Delta B(i+1, j) &= \varepsilon_0 \cdot (1 + \varepsilon_r) \cdot B(i, j) \cdot \Delta t \\ \Delta B(i, j-1) &= \varepsilon_0 \cdot B(i, j) \cdot \Delta t \\ \Delta B(i, j+1) &= \varepsilon_0 \cdot B(i, j) \cdot \Delta t\end{aligned}$$

(i, j) denotes the cell in the i-th row and j-th column.

The differential equations are solved with Euler's scheme. The time step for the calculations is $\Delta t = 0.01$.

Parameters

Par.	value	unit	meaning
r	54.75	1/y	growth in old forest
K	5000	bugs/cell	capacity of bugs
μ	36500	bugs/cell/y	predation by birds
M	500	bugs/cell	bug density controlling predation
α	0.15	1/y	aging factor of forest
β	1.7	1/y	damage of forest
ε_0	1.0	1/y	maximum part of bugs to diffuse
ε_r	0.9	-	parameter for direction (wind force)

The simulation shows the number of bugs per cell. It takes several hundred generations until the dynamical state of the system appears. Unfortunately this example is very slow due to the numerical solver of the differential equations.

2.5.7 Diffusion

Demonstration of diffusion. The simulation should be started with a random filling or a small filled area (circle or square). The colortable is fixed to white black. Filling can be done by a left mouse click

2.5.8 Advection

Demonstration of advective transport. The simulation should be started with a small filled area (circle or square). The colortable is fixed to white black. Filling can be done by a left mouse click. Three scenarios are implemented:

1. **1D advection (1 cell per time step):** simulation of advection without numerical diffusion. The advection velocity is set to one cell per time step in x direction
2. **1D advection (0.1 cell per time step):** simulation of advection demonstrating the effect of numerical diffusion. The advection velocity is set to 0.1 cell per time step in x direction.
3. **2D advection (0.1 cell per time step):** simulation of advection demonstrating the effect of numerical diffusion. The advection velocity is set to 0.1 cell per time step in north-east direction.

2.5.9 Leopard

The model describes a mechanism that can describe how the coat pattern of a leopard can develop through diffusion processes. The model considers two substances with different properties.

The Activator: Substance that produces the expression of a characteristic (pigment production) with the following characteristics:

- autocatalytic (self-reinforcing)
- generates inhibitor
- diffuses slowly (localized)

The Inhibitor: Substance that suppresses expression of the characteristic

- suppresses activator production
- diffuses quickly (long-range)

$$\frac{\partial a}{\partial t} = \frac{a^2}{b} - \mu_a \cdot a + \varepsilon_a + D_a \frac{\partial^2 a}{\partial x^2} \quad \text{Activator}$$

$$\frac{\partial b}{\partial t} = a^2 - \mu_b \cdot b + \varepsilon_b + D_b \frac{\partial^2 b}{\partial x^2} \quad \text{Inhibitor}$$

$\mu_a = 1.0$ Decay rate of the Activator

$\mu_b = 1.0$ Decay rate of the Inhibitor

$\varepsilon_a = 0.01$ Basic production rate of the Activator

$\varepsilon_b = 0.01$ Basic production rate of the Inhibitor

$D_a = D_b/8.0$ Diffusion constant of the Activator (fixed ratio)

$D_b = 4.0$ or 8.0 Diffusion constant of the Inhibitor

There are two models selectable with $D_b = 4.0$ and $D_b = 8.0$.

2.5.10 [Color Table]

The different automata look nicest with their default color table, but the colortable can be changed for some automata and is displayed in the color bar. The automata with discrete states are best visualized by the discrete color table. The number of colors displayed in the color bar corresponds to the number of states of the active automata.

2.5.11 Cells per row

The number of cells per row can be selected. The automata field is then built quadratic. The higher the number the slower the simulation runs.

2.5.12 View of the world

The world can either be assumed as closed (Torus) in this case the first row is neighbour of the last one and the first column is neighbour of the last column.

If the case of a plain world boundary cells have less neighbours.

2.5.13 [Show]

If **[Show]** is switched on, every generation is shown, if switched off the picture is not actualized until **[Show]** is switched on again. This is useful if only higher generation numbers are of interest because the simulation becomes quite faster without displaying.

2.5.14 [Run]

[Run] starts the simulation. The generation is displayed in the info window. The simulation can be stopped by **[Interrupt]**.

2.5.15 [Step]

[Step] calculates the next generation

2.5.16 [Interrupt]

[Interrupt] stops the simulation.

2.5.17 [Clear]

[Clear] sets all cells to zero.

2.5.18 Setting the initial states

The initial state of the automat can either be set by mouse within the automat field. Depending on the number of states the following settings are possible

- left mouse click sets cell to 1
- <Shift>+ left mouse click sets the cell to 2
- <Strg>+ left mouse click resp. <ctrl>+ left mouse click sets the cell to 3
- middle or right mouse click resets the cell to 0

It is possible to save a configuration by **[Save]** in a file. These files get automatically the extension `lif` and are stored normally in `<Path to Mathtools>/mftfiles`. The configuration can be loaded by **[Load]**

[Randomfill]

[Randomfill] fills the field at random depending on the number of possible states.

[Fill]

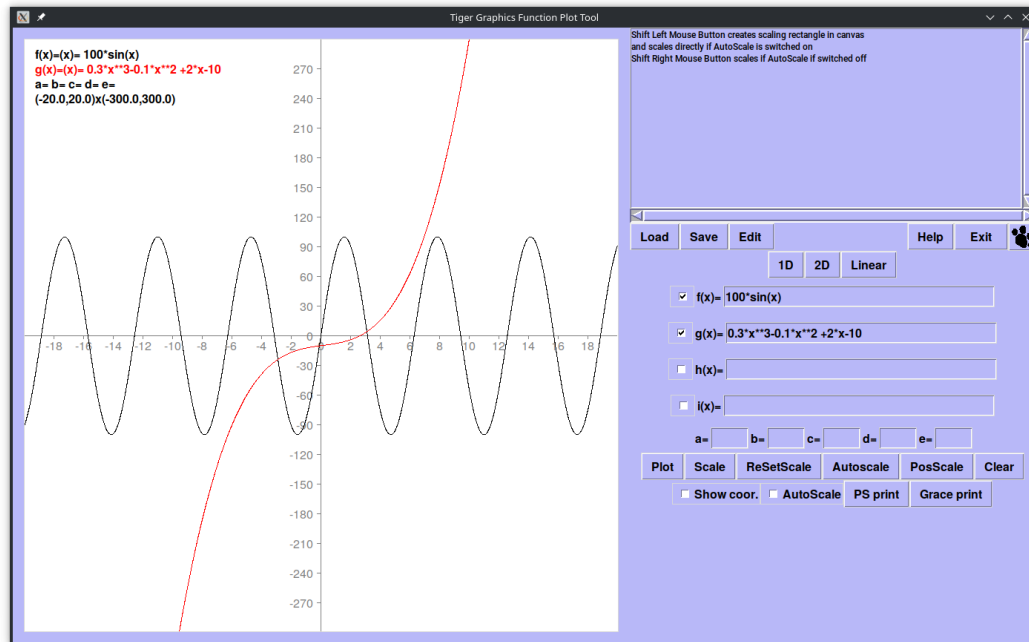
All cells are filled with the selected fill value.

2.5.19 PS print/Print

A Postscript output is generated and stored in `<Path to Store Directory>/plots`. The PS-files are named automatically.

2.6 Plotter

This tool allows the visualization of functions. In the **[1D]** mode up to three one dimensional functions of the form $f(x) = \dots$ can be plotted. The function is only plotted if the checkbox on the left of the definition is switched on.



In the **[2D]** mode a parametrized two dimensional function of the form $x(t) = \dots$ $y(t) = \dots$ can be plotted.

The following math functions can be used within the formulas:

```
acos asin atan cos cosh exp log log10
pow sin sinh sqrt tan tanh abs
```

The formulas can be defined with up to 5 parameters (a, b, c, d, e).

2.6.1 [Plot]

The defined function is plotted according to the settings of the independent variable x **[1D]** or t **[2D]**. The axes can either be set by the sliders or set by **[Scale]** or **[AutoScale]**

2.6.2 [Scale]

The dimension of the t, x and y values can be set. **[Scale]** refreshes the plot with the just defined range.

2.6.3 [ReSetScale]

This button resets the the range of the plotting area to the initial values.

2.6.4 [Autoscale]

The maximum values of y **[1D]** resp. x and y **[2D]** are evaluated and the plot is adapted to that values according to the settings of the independent variable.

2.6.5 [PosScale]

The x and y range is shifted to the positive half of the plane.

2.6.6 [Clear]

[Clear] clears the output window.

2.6.7 Coordinates

A left mouse click in the output windows shows a horizontal line within the output window at the actual y-coordinate and its value, a right mouse click shows a vertical line within the output window at the actual x-coordinate and its value.

Show Coor.

If **[Show Coor.]** is switched on the coordinates are shown at the mouse pointer.

AutoScale

If **[AutoScale]** is switched on the definition of a scaling/zooming area by `Shift-Left Mouse Button` is directly applied. Otherwise `Shift-Right Mouse Button` will initiate the scaling. If a scaling rectangle is drawn but should not apply, it can be erased by pressing the `<Escape>`-Button on the keyboard.

2.6.8 [Load] , [Save] & [Edit]

Equations, parameter values and coordinates can be saved in a file. These files get automatically the extension `tgp [1D]` or `tgt [2D]` and are stored normally in `<Path to Mathtools>/fracfiles`. Before loading a file the mode (**[1D]** or **[2D]**) must be selected. After loading the file the equations, parameter values and the stored coordinate values are set. The files can also be edited to set f.e. coordinate values not provided by default. The editor must be set in **[Custom]** (2.1).

2.6.9 PS print/Print

A Postscript output is generated and stored in `<Path to Store Directory>/plots`. The PS-files are named automatically.

2.6.10 Grace print

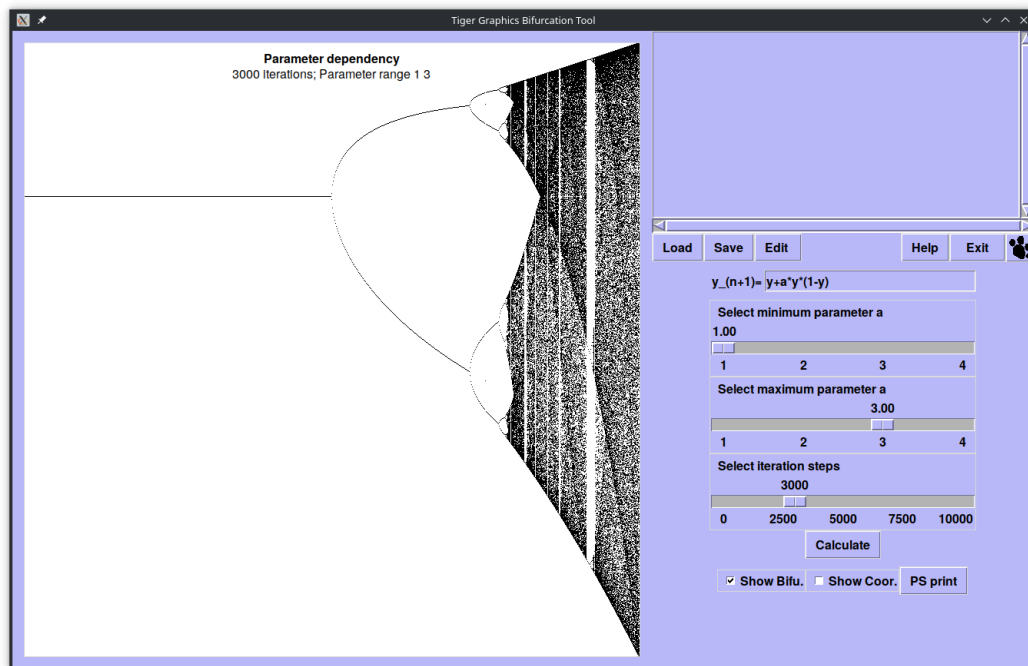
An **xmGrace** output is generated and stored in `<Path to Store Directory>/plots`. The xmgr-files are named automatically.

2.7 Parabola

The parabola equation

$$x_{n+1} = r \cdot x_n \cdot (1 - x_n)$$

converges, shows cycles or chaos depending on the parameter r .

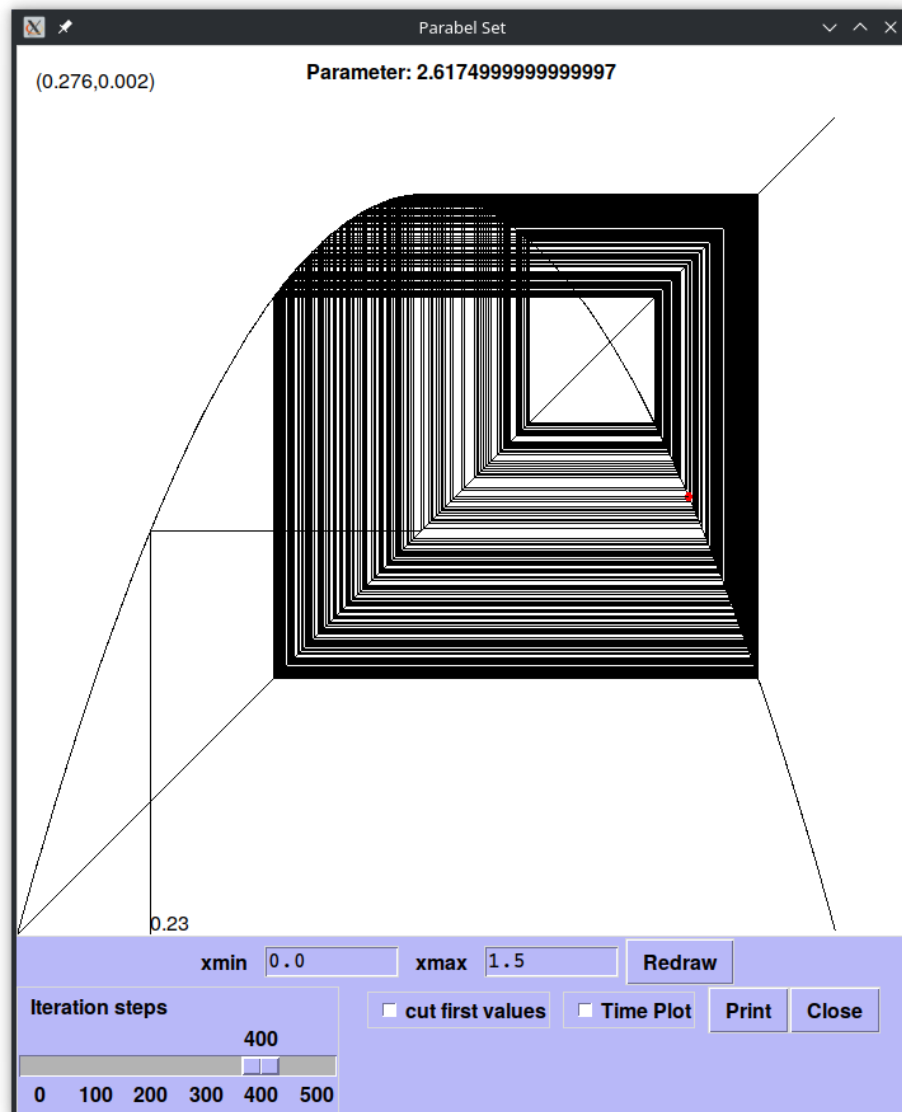


2.7.1 [Calculate]

Shows the location of the limit points depending on the parameter r . Clicking with the left mouse button into the picture shows the parabola of the selected value of r :

$$f(x) = r \cdot x \cdot (1 - x)$$

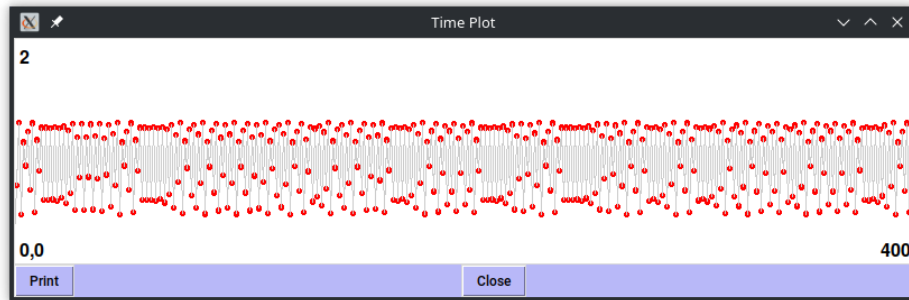
2.7.2 Single Parameter Parabola



A left mouse click starts the iteration within the parabola (initial value). With middle mouse clicks the iteration can be continued step by step until the selected number of iterations is reached. A right mouse click shows all iteration steps at once depending on the selected number of iterations.

If **[cut first values]** is switched on the initialization is cut off: only the limit point resp. cycle resp. chaos is shown.

If **[time plot]** is switched on the development of the iteration is shown in a separate window.



Remark: If the parametrization leads to a limit point switching [cut first values] on is not sensible because the limit point cannot be seen.

2.7.3 PS print/Print

A Postscript output is generated and stored in
 <Path to Store Directory>/plots. The PS-files are named automatically.

2.7.4 Misc

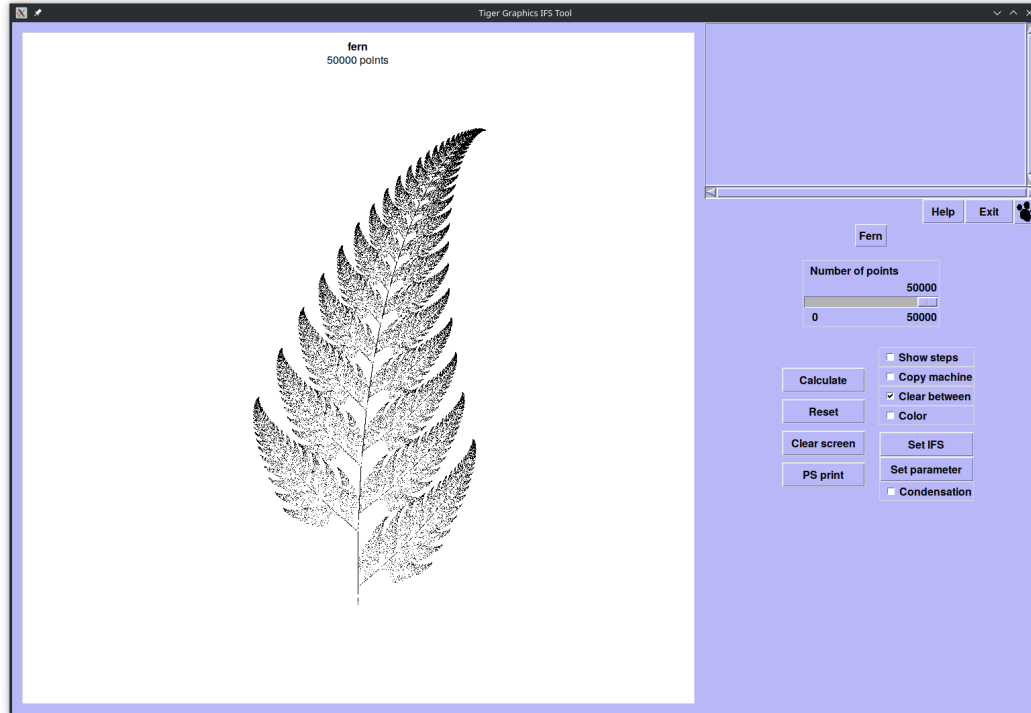
The tool allows also the visualization of

$$x_{n+1} = x_n + r \cdot x_n \cdot (1 - x_n)$$

which is the discretization of the logistic differential equation

$$\dot{x} = x \cdot (1 - x)$$

2.8 IFS - Iterative function systems



Iterative function systems are generators for self similar images. Further information on IFS can be found in Barnsley (2000).

This tool allows to define up to 4 affine maps per image. Predefined sets of maps are available for

- Fern leaf
- Highway dragon
- Maple leaf
- Simple tree
- Phytagoras tree
- Sierpinski's triangle
- Chess board²

2.8.1 [Calculate]

[Copy machine] switched off

Starting from an initial point the chaos game is started. The number of points to be drawn can be selected. Due to the accuracy in the random function the algorithm runs into a cycle. Such only a few number of points can be distinguished.

[Copy machine] switched on

A quadrangle is mapped by the given affine maps. Pressing calculate gain maps the new created quadrangles again and so on. Because of the increasing number of quadrangles this process become slower and slower. Such only 8 iterations are possible by the machine. **[Reset]** resets this process. If **[Color]** is selected each step is plotted in a different color. If **[Clear between]** is selected only the quadrangles of the last step are printed.

²The Chessboard is defined by 5 affine maps. The last one is treated as condensation!

2.8.2 [Reset]

Resets the iteration of the copy machine (see 2.8.1)

2.8.3 [Clear screen]

All printed stuff is deleted. The maps still exist.

2.8.4 PS print/Print

A Postscript output is generated and stored in
`<Path to Store Directory>/plots`. The PS-files are named automatically.

2.8.5 [Set IFS]

A reference rectangle occurs in the picture window. By left mouse click up to four affine maps can be defined. The first click defines the image point of the upper left corner, the second of the lower left corner and the third the image point of the lower right corner. The resulting quadrangle is plotted. This process can be repeated three times.

The last map is additionally stored as fourth map. It is only evaluated if

- four different maps are defined, or
- **[Condensation]** is switched on

2.8.6 [Set Parameters]

The parameters of the actual IFS are shown and can be modified by

- selecting another predefined IFS, or
- loading an IFS which has been saved further, or
- defining a new IFS by **[Set IFS]**

Every row represent a map. The parameters are defined as follows:

$$f_i \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} a_i & b_i \\ c_i & d_i \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} + \begin{pmatrix} e_i \\ f_i \end{pmatrix}, \quad i = 1, \dots, 4$$

The values w_i in the last column give the probabilities for selecting this map by the chaos game. If the sum of the first probabilities exceed 1, the following maps are not evaluated. If **[Condensation is active]**, the last map is evaluated as condensation independent of the probabilities. The probability of the predefined maps are defined by the determinant of the matrix. If an IFS is defined by **[Set IFS]** the probabilities are equally distributed. The values can be saved in a file. The files holding IFS-data get the extension `.ifs` and are stored in `<Path to Mathtools>/mftfiles` by default.

Bibliography

[Barnsley 2000] BARNSELY, M.: *Fractals everywhere*. Morgan Kaufmann, 2000. – 534 S